with a certain law, by a particular element $a'$ corresponding to it (which can be contained in $A$ or not); one calls such a law a substitution, and says that this substitution takes the element $a$ into the element $a'$, and similarly takes the system $A$ into the system $A'$ of elements $a'$. [Dedekind's footnote:] Already in the third edition of this work (1879, remark on p. 470) it is stated that no thought of any kind is possible without the capacity of the mind [Fähigkeit des Geistes] to compare a thing $a$ with a thing $a'$, or to relate $a$ to $a'$, or to allow an $a'$ to correspond to $a$—and that the entire science of numbers also rests on this capacity. The elaboration of this thought has subsequently been published in my article, 'Was sind und was sollen die Zahlen?' (Brunswick, 1888); the notation employed there for mappings and their composition differs slightly in external features from the one used here.

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**G. LETTER TO HEINRICH WEBER**

(24 JANUARY 1888)

The following extract from Dedekind's correspondence with Weber was published in *Dedekind 1930–2*, Vol. iii, pp. 488–90. (Other letters to and from Weber are published in *Dedekind 1930–2* and in the appendices to *Dugac 1976*.) The translation is by William Ewald.

... I am delighted that you take such an interest in my article on numbers; not many do so. Cantor has called my attention to the fact that he already mentioned the difference between the finite and the infinite in 1877 (*Crelle*, Vol. 84, p. 242 [Cantor 1878]), but says that he does not intend to claim priority. One could discuss this matter at length; he is right to a certain extent, but in 1882 he doubted the possibility of a simple definition, and was greatly surprised when I, as a result of his doubts, and at his request, communicated my definition to him; sometimes one possesses something without appreciating its value and significance. But I too have no wish to get involved in a conflict over priorities.—I have repeatedly read and thought through your remarks and suggestions; but whether they would lead to an essential simplification and shortening is difficult to say without seeing the new version in full detail. Besides, I must confess to you that I still regard the ordinal number and not the cardinal number (Anzahl) as the original number-concept. It would perhaps have been better had I not mentioned these names (ordinal, cardinal) in my paper, since they are used in a different sense in ordinary grammar. My ordinal numbers, the abstract elements of the ordered simply-infinite system, have of course nothing to do with the adjectival form of the so-called (in grammar) ordinal numbers, from which form one could extract an argument for the conceptual priority of the cardinal numbers (Anzahlen). This adjectival form is also used where there is no question of an ordering (and consequently of my ordinal numbers), e.g. when one speaks of the fifth part of an interval. I hold the cardinal number (Anzahl) to be only an *application* of the ordinal number, and in our *zählende Gegenstände* ['counting'] too one reaches the concept five only via the concept four. But if one were to take your route—and I would strongly urge that it be explored once to the end—then I would advise that by number (Anzahl, cardinal number) one understand not the *class* itself (the system of all finite systems that are similar to each other) but something *new* (corresponding to this class) which the mind *creates*. We are a divine race and undoubtedly possess creative power, not merely in material things (railways, telegraphs) but especially in things of the mind. This is precisely the same question that you raise at the end of your letter in connection with my theory of irrationals, where you say that the irrational number is nothing other than the cut itself, while I prefer to create something *new* (different from the cut) that corresponds to the cut and of which I say that it brings forth, creates the cut. We have the right to ascribe such a creative power to ourselves; and moreover, because of the similarity [Gleichartigkeit] of all numbers, it is more expedient to proceed in this way. The rational numbers also produce cuts, but I would certainly not call the rational number identical with the cut it produces; and after the introduction of the irrational numbers one will often speak of cut-phenomena with such expressions, and ascribe to them such attributes, as would sound in the highest degree peculiar were they to be applied to the numbers themselves. Something quite similar holds for the definition of cardinal number (Anzahl) as a class; one will say many things about the class (e.g. that it is a system of infinitely many elements, namely, of all similar systems) that one would apply to the number only with the greatest reluctance; does anybody think, or won't he gladly forget, that the number four is a system of infinitely many elements? (But that the number four is the child of the number three and the mother of the number five is something that nobody will forget.) For the same reason, I always considered Kummer's *creation* of the ideal numbers to be thoroughly justified, if only it were rigorously carried out. Whether in addition the language of symbols [Zeichen-sprache] suffices to designate uniquely each individual that is to be created, is not important; it always suffices to designate the individuals that appear in any (limited) investigation...