University of Bristol
Mathematics Exam
Practice paper

Instructions

- You have **24 hours** to complete this exam.
- Calculators are permitted.
- Show your working clearly. Answers without working will not be given full credit.
- The marks for each question part are shown in brackets.

1. Let $w_1 = 3 + 4i$ and $w_2 = -1 + 2i$.

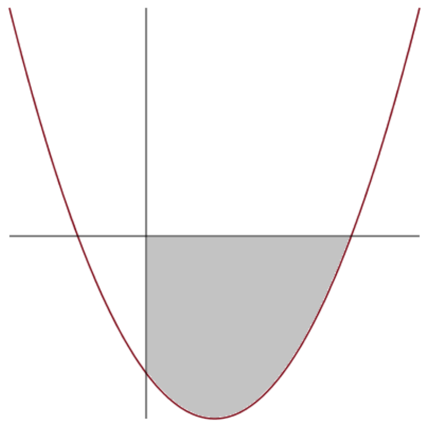
   (a) Find the following, giving your answers in the form $a + bi$.

      i. $w_1^* w_2$ **[2]**

      ii. $\dfrac{w_1 - 3w_2}{w_2}$ **[2]**

   (b) Write down a quadratic equation with real coefficients that has w_1 as a root. **[2]**

2. The graph of the function $y = x^2 - 2x - 3$ is sketched below. Find the area of the shaded region. **[4]**



3. Given that $\mathbf{i}$ and $\mathbf{j}$ represent two perpendicular unit vectors,

   (a) Find the magnitude of the vector $-8\mathbf{i} + 15\mathbf{j}$ **[2]**

   (b) Find the angle θ between the vector $-8\mathbf{i} + 15\mathbf{j}$ and the vector $\mathbf{j}$, giving your answer in degrees to 2 decimal places. **[4]**

4. In 2005 there were 20 puffins on Lundy island. Counting $t = 0$ as the year 2005, after t years the number of puffins on the island, P, is modelled by the equation

$$\frac{dP}{dt} = 0.2P$$

   (a) Find an equation for P in terms of t. **[5]**

   (b) Hence estimate, to the nearest hundred, how many puffins there will be on Lundy island in the year 2030. **[2]**

   (c) Is this model suitable for predicting how many puffins will be on the island in the year 2100? Give a reason for your answer. **[1]**

5. Let $w = 5e^{3\pi i/4}$ and $z = 4e^{\pi i/2}$.

   (a) State the value of $|wz|$. **[1]**

   (b) State the value of $\arg\left(\dfrac{w}{z}\right)$ **[1]**

   The complex number v satisfies $v^2 = z$, with z as above.

   (c) Write down the possible values of v in the form $re^{i\theta}$ with $r > 0$ and $-\pi < \theta \leq \pi$. **[2]**

6. The matrices A and B satisfy the equation

$$AB = I - 2A$$

where I is the identity matrix and $B = \begin{pmatrix} 5 & -4 \\ -3 & 0 \end{pmatrix}$.

Find A. **[3]**

7. It is given that x and y are related by the ordinary differential equation

$$\frac{dy}{dx} = 8 - \frac{3y}{1+x}$$

and that when $x = 0$, $y = -14$.

Find the value of y when $x = 1$. **[6]**

8. Find a vector that is perpendicular to both $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}$. **[3]**

9. A complex number z has modulus 1 and argument θ.

   (a) Show that

$$z^n + \frac{1}{z^n} = 2\cos(n\theta)$$

   for any integer n. **[3]**

   (b) Hence show that $\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$ **[6]**

10. (a) Find $\int 9x^5\sqrt{x^3+2}\,dx$ **[5]**

    (b) Find $\int_{\ln 2}^{\ln 4} 2ue^{2u}\,du$, giving your answer in the form $a\ln 2 + b$, where a and b are constants to be found. **[4]**

11. Consider the simultaneous equations

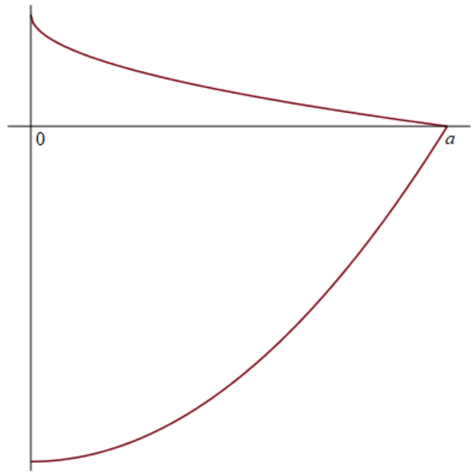
$$3x + 4y = 6$$
$$-x + 4y = 19$$

    (a) Write these equations as a matrix equation $A\mathbf{x} = \mathbf{b}$. **[2]**

    (b) Find the determinant of the matrix $\begin{pmatrix} 6 & 4 \\ 19 & 4 \end{pmatrix}$. **[2]**

    (c) Use Cramer's rule to solve the simultaneous equations. You must show detailed reasoning. **[3]**

12. The sketch below shows the curves $y = A - 2\sqrt{x}$ and $y = x^2 - B$ for x in the range $0 \leq x \leq a$, where A, B, and a are all positive constants. The sketch is not drawn to scale. Both curves meet the x-axis at $x = a$.



(a) Find the area of the finite region bounded by the y-axis and the two curves, giving your answer in terms of A. **[7]**

The area of this region is 504.

(b) Find the value of A. **[3]**

(c) Hence find the values of B and a. **[1]**

13. The points $A(4, 5, -1)$, $B(4, -5, 8)$, $C(6, 1, -1)$, and $D(6, 11, -10)$ are the four corners of a parallelogram. Find the area of the parallelogram $ABCD$.

[5]

14. (a) Sketch an Argand diagram showing the locus of points satisfying the equation

$$|z + 2| = 2.$$

[2]

(b) Given that there is a unique complex number w that satisfies both

$$|w + 2| = 2$$

and

$$\arg(w - k) = \frac{3\pi}{4},$$

where k is a positive real number,

i. Find the value of k. **[2]**

ii. Express w in the form $r(\cos\theta + i\sin\theta)$, giving r and θ to 2 significant figures. **[4]**

15. A particle moves around a fixed point O with damped harmonic motion. The displacement in metres of the particle from O at time t seconds is denoted by x, where x satisfies the equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

(a) Find the general solution to this differential equation. **[3]**

When $t = 0$ the particle is a distance of 15 m from O and is moving with velocity 5 ms^{-1}.

(b) Find an expression for x in terms of t. **[3]**

(c) Briefly explain what happens to the particle as t increases. **[1]**

A second particle has displacement u from O at time t given by the equation

$$\frac{d^2u}{dt^2} + 6\frac{du}{dt} + 9u = e^{-3t}$$

(d) Find the general solution to this equation, expressing u in terms of t. **[6]**

END OF EXAM.